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Enhancement of Natural Convection by Eccentricity of Power Cable Inside Underground Conduit

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Nomenclature

D_h	=	hydraulic diameter
e	=	eccentricity
Gr	=	Grashof number, Ra/Pr
g	=	gravitational acceleration
k	=	thermal conductivity
l	=	radial distance between the inner and outer cylinders
Nu	=	Nusselt number, $h^* D_h^* / k_{\text{ref}}^*$
Pr	=	Prandtl number, υ^*/α^*
q	=	heat flux
Ra^o	=	modified Rayleigh number, $g^*\beta^* D_h^{*4} q_w^* / \upsilon^*\alpha^* k^*$
T	=	temperature
T_b	=	bulk temperature
$T_{s,\max}$	=	maximum cable surface temperature
V	=	velocity
(x, y, z)	=	Cartesian coordinates, $(x^*, y^*, z^*)/D_h^*$
α	=	thermal diffusivity
β	=	thermal expansion coefficient
θ	=	nondimensionalized temperature, $(T^* - T_{\text{ref}}^*)/q_w^* D_h^*/k^*$
υ	=	kinematic viscosity
ϕ	=	azimuthal angle

Subscripts

b	=	bulk
i	=	inner cylinder
l	=	local
o	=	outer cylinder
ref	=	reference state (at atmospheric pressure and room temperature)
w	=	wall

Superscripts

$-$	=	averaged quantity
$*$	=	dimensional quantity

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Introduction

PLACING the power cable underground is a current engineering tendency due to the limited available space, particularly in city areas and industrial zones. The power cable (inner cylinder) is placed inside a concrete conduit (outer cylinder) buried underground. The configuration of this layout is an annulus between two horizontal, highly eccentric cylinders as schematically shown in Fig. 1. Heat is generated due to the electrical resistance of the power cable, and the heat dissipation process in the annulus relies on the natural convection heat transfer from both open ends of the conduit, which penetrate onto the manhole surfaces. As described in our previous study,[1] the geometric configuration of this eccentric annulus between two horizontal cylinders and its associated thermal boundary conditions lead to a three-dimensional, noncavity-type problem, which was seldom reported on in published work. An up-to-date review on the natural convection heat transfer in the annulus between two horizontal cylinders for two-/three-dimensional and concentric/eccentric configurations may be found in our previous work[1] and is not repeated here.

It was found from our previous work[1] that the highest temperature of the power cable is always located at the contacting point of the cable and the concrete conduit. The cause can be apparently understood from the azimuthal distributions of the local Rayleigh number, which is defined by

$$Ra_l(\phi, z) = \frac{g^*\beta_{\text{ref}}^*\left[T_i^*(\phi, z) - T_o^*(\phi, z)\right]l^{*3}}{\upsilon_{\text{ref}}^*\alpha_{\text{ref}}^*} \tag{1}$$

where l^* is the radial distance between the inner (cable) and outer (concrete conduit) surfaces, with the pole located at the center of the inner cylinder, at a given azimuthal angle ϕ and longitudinal position z. For the ordinary configuration of the cable inside an underground conduit, the cable (inner cylinder) lies on the bottom of the concrete conduit (outer cylinder). Clearly, the l^* value approaches zero as ϕ moves to the contacting point. As a result, the Rayleigh number $Ra_l(\propto l^{*3})$ in the neighborhood of the contacting point drops steeply to very small values. By definition, the Rayleigh number is equal to the Grashof number times the Prandtl number, and the Grashof number provides a measure of the ratio of the buoyancy force to the viscous force acting on the fluid. At small Rayleigh number Ra_l, the local heat transfer is mainly through the heat conduction process, and this leads to a poor heat dissipation rate.

It is known[2–6] that natural convection heat transfer rate in the annulus between two horizontal cylinders can be enhanced by

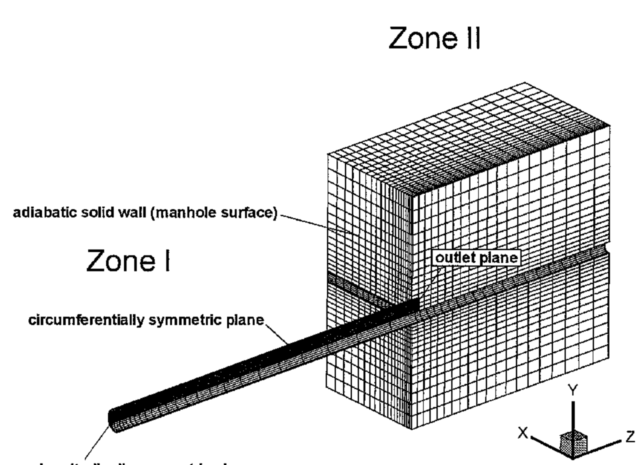

Fig. 1 Computational domain and zonal grid distribution. Origin (0, 0, 0) is located at the top point (for $e = 0.5$) or the bottom point (for the other cases) of the inner cylinder on the longitudinally symmetric plane.

changing its eccentricity. A typical example is the numerical study of Hirose et al.[5] In their work, a two-dimensional steady natural convection problem within a horizontal eccentric annulus, whose surfaces were maintained at two constant temperatures, was studied for the medium of air with Rayleigh numbers ranging from 10^4 to 10^5. Their study revealed that the enhancement of the natural convection heat transfer rate in the annulus can be achieved by adjusting the eccentricity and the radius ratio of the inner to outer cylinders. However, very few three-dimensional studies on this topic are found in the published literature. The one studied numerically by Vaidya and Shamsundar[6] is a cavity configuration bounded by two insulated end walls between two horizontal isothermal eccentric cylinders, which is quite different from the configuration of the underground conduit of an electrical power cable to be investigated here.

It is known that the existence of higher local temperatures on the power cable will lead to earlier deterioration of the cable than its designed usage time. The scope of this work is to study how and what level of the maximum temperature decrement can be achieved by adjusting the eccentricity of the power cable inside an underground conduit.

Results and Discussion

Because the thermal boundary conditions at the walls are specified in terms of heat fluxes instead of temperatures in the present work, a modified Rayleigh number is defined as follows:

$$Ra^o = \frac{g^* \beta^*_{\text{ref}} D^{*3}_h}{\upsilon^*_{\text{ref}} \alpha^*_{\text{ref}}} \left(q^*_w D^*_h \big/ k^*_{\text{ref}} \right) \tag{2}$$

The case of $Ra^o = 10^6$, which leads to distinct temperature variations on the power cable surface, is calculated for demonstration. See Ref. 1 for information and discussion of the numerical aspects.

The definition of eccentricity is shown in Fig. 2. Six configurations with various eccentricities, $e = 0.5$, 0.45, 0.4, 0.25, 0, and -0.25, are examined to investigate the effect of eccentricity on natural convection heat dissipation inside the cable conduit. Note that $e = 0.5$ denotes the case of the ordinary configuration of the power cable inside an underground conduit, whereas $e = 0$ is the case of the concentric annulus.

Figure 3 shows the axial distributions of the maximum cable surface temperatures $T_{s,\max}$, which are always located at the bottom of the cable. The trend in Fig. 3. clearly reveals that an effective decrement in $T_{s,\max}$ can be achieved by reducing the eccentric level of the inner cylinder in the annulus. For example, about an 8-K decrement in $T_{s,\max}$ can be achieved even with a small elevation ($10\%\ r_i$) of the inner cylinder from the ordinary configuration of the power cable inside the underground conduit. Furthermore, about a 20-K decrement in $T_{s,\max}$ can be achieved through the use of the optimal configuration ($e = 0$). This result is rather different from two-dimensional results, such as the work of Hirose et al.[5] Although their work was based on two constant surface temperatures with hot side on the inner cylinder, their results (see Fig. 5c in Ref. 5) showed that the optimal configuration was far different from the concentric one ($e = 0$) in the Rayleigh number range of 10^4–10^5. This is because no axial flow, which can be clearly observed in the present flow situation,[1] occurs in two-dimensional (only radial and angular) simulation. Obviously, the axial flow motion plays an important role in the process of natural convection heat transfer for the present configuration.

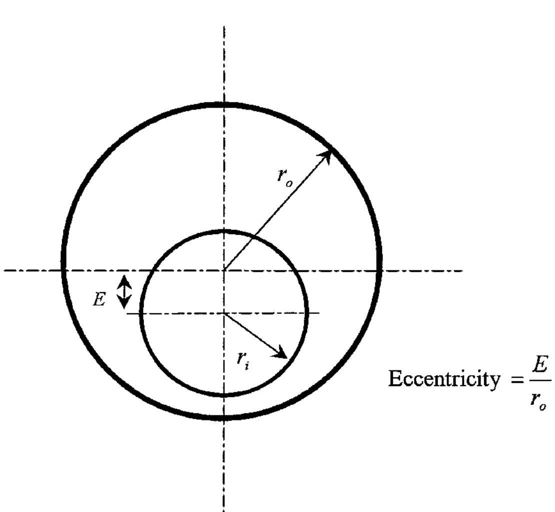

Fig. 2 Definition of eccentricity.

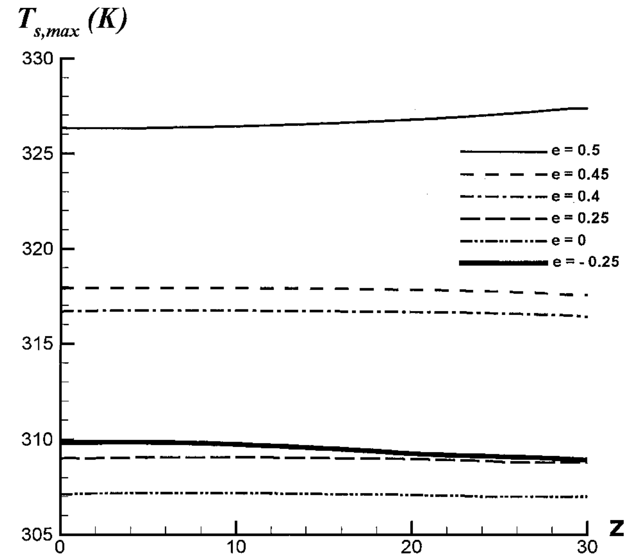

Fig. 3 Axial distributions of the maximum cable surface temperature for the six examined configurations.

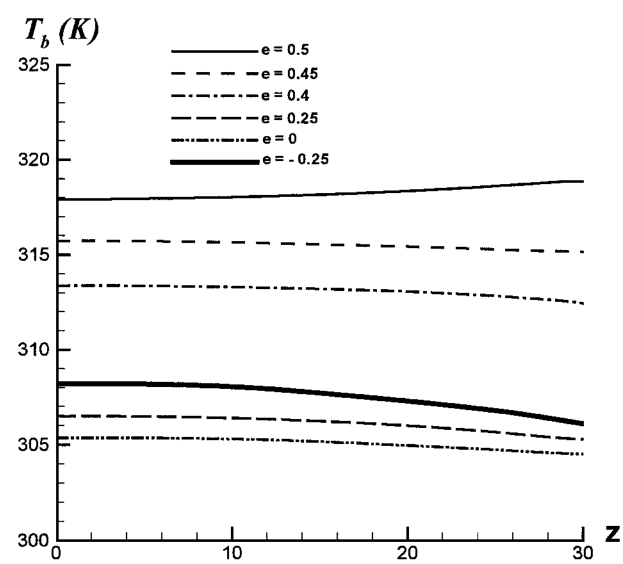

Fig. 4 Axial distributions of the bulk temperature for the six examined configurations.

Figure 4 shows the axial distributions of the bulk temperature T_b, which for the six examined configurations is defined as[7]

$$\theta_b(z) = \frac{\oiint_A \theta(r, \phi, z)\rho |\boldsymbol{V} \cdot \mathrm{d}\boldsymbol{A}|}{\oiint_A \rho |\boldsymbol{V} \cdot \mathrm{d}\boldsymbol{A}|} \tag{3}$$

The trend for T_b is similar to that for $T_{s,\max}$, but the differences among various T_b are smaller than those among various $T_{s,\max}$. This is because the area integration in Eq. (3) alleviates the roles of $T_{s,\max}$ and, therefore, leads to the results shown in Fig. 4.

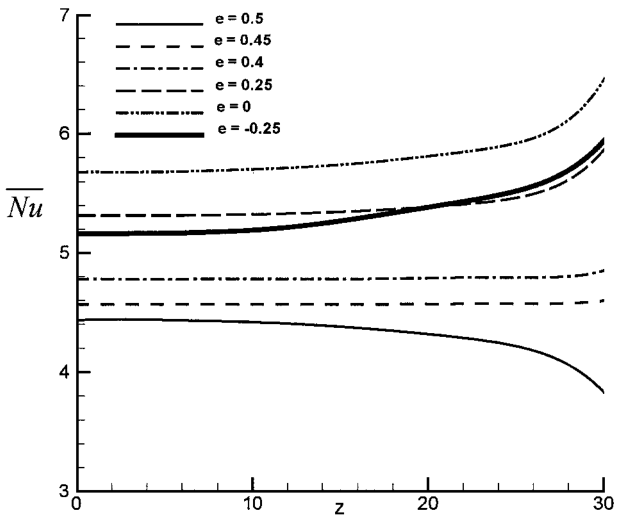

Fig. 5 Axial distributions of the averaged Nusselt number on the inner cylinder for the six examined configurations.

Figure 5 compares the averaged Nusselt number distributions on the cable surfaces of the six examined configurations. Here the averaged Nusselt number is defined by

$$\overline{Nu}(z) = \frac{1}{\pi}\int_0^{\pi} Nu(\phi, z)\,\mathrm{d}\phi \tag{4}$$

where

$$Nu(\phi, z) = 1/[\theta_w(\phi, z) - \theta_b(z)] \tag{5}$$

Note that the Nusselt number defined on the outer (concrete conduit) surface is identically zero due to the specified adiabatic boundary condition. Except for the case of $e = 0.5$, the Nusselt number $\overline{Nu}$ for the other five cases increase toward the open end.

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Computations of Incompressible Flows with Natural Convection Using Pseudocompressibility Approach

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Introduction

In recent years, there has been renewed interest in the pseudocompressibility method for computing incompressible flows due to its numerous advantages.[1] This method, originally proposed by Chorin,[2] has found its application in many areas pertaining to isothermal flows.[1,3–7] However, to our knowledge, there is no detailed investigation about the applicability of the pseudocompressibility method to heat transfer problems.

The primary difficulty in computing incompressible flows is in finding a satisfactory way to link changes in the velocity field to changes in the pressure field. This link must be accomplished to ensure the divergence-free velocity field. Among the commonly used methods for handling the velocity pressure coupling for three-dimensional problems are the pressure-based method (PBM) and the pseudocompressibility method (PCM). The basic idea in the PBM is to formulate an elliptic equation for pressure correction to update the pressure and maintain a divergence-free velocity field. The PBM, although widely used in industry, is complex computationally in the treatment of boundary conditions.[8] On the other hand, in the PCM, an artificial compressibility term is introduced in the continuity equation, which makes the system of equations strongly coupled and hyperbolic–parabolic in nature. Because of this direct coupling between the continuity and the momentum equations the PCM has been found to yield better convergence than the PBM.[8]

In the present work, the pseudocompressibility approach has been extended to compute heat transfer problems for both laminar and turbulent flow situations. A few standard natural convection test cases are evaluated to demonstrate the capability of the present formulation, namely, laminar flow in a two-dimensional differentially heated cavity,[9] a concentric annulus,[10] and a three-dimensional thermal cavity,[11] and turbulent flows inside a differentially heated cavity.[12]

Mathematical Formulation

The governing equations considered here are the time-dependent incompressible Reynolds averaged Navier–Stokes equations with the shear stress transport (SST) turbulence model,[13]

$$\frac{\partial W}{\partial t} + \frac{\partial(F^c - F^v)}{\partial x} + \frac{\partial(G^c - G^v)}{\partial y} + \frac{\partial(H^c - H^v)}{\partial z} = S$$

where

$$W = [p, u, v, w, T, k, \omega]^T$$

$$F^c = \left[\beta_{ps}u,\ u^2 + p + 2/3k,\ uv,\ uw,\ uT,\ uk,\ u\omega\right]^T$$

$$G^c = \left[\beta_{ps}v,\ uv,\ v^2 + p + 2/3k,\ vw,\ vT,\ vk,\ v\omega\right]^T$$

$$H^c = \left[\beta_{ps}w,\ uw,\ vw,\ w^2 + p + 2/3k,\ wT,\ wk,\ w\omega\right]^T$$

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